Wind Energy Math Calculations

Estimating the Height of a Wind Turbine Using Triangles

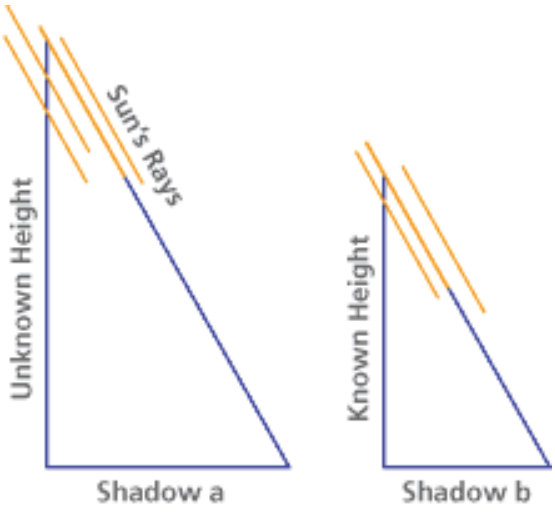
If you have seen a real utility-scale wind turbine, you know that they are REALLY TALL! But how tall are they exactly? Of course no measuring tape is long enough to reach the top of a wind turbine, so if you want to find the height, you'll have to use another method.

There are a few simple ways to measure the height of a tall object pretty accurately, but you'll have to use some math! If you don't have a wind turbine nearby to try out these measurements, you can also use these to measure trees, etc.

<u>Method 1: Measuring Height Using Shadows and Similar Triangles</u>

You'll need to bring a few "tools" with you for this method.

- A Yardstick or another object of known height
- A calculator
- A long string can be helpful for measuring long distances
- Sunshine! You'll be using shadows so you need a sunny day



1. Start by measuring the length of the shadow of the wind turbine. If you have a long string, that can be helpful, but you can also "pace" the length and measure how long your pace is (then multiply that length by the number of steps you took).

2. Next measure the shadow cast by the yard-stick (or other object of known length).

3. Since you know the height of the yardstick and the length of the two shadows, you can set up a simple **ratio** to find the height of the wind turbine.

It is important to take these two measurements at the same time of day so that the sun's rays can be assumed to be parallel. Since the sun's rays are parallel, the two triangles are **similar** and therefore proportional. This is the ratio of the two triangles:

$$\frac{\textbf{Unknown Height}}{\textbf{Shadow a}} = \frac{\textbf{Known Height}}{\textbf{Shadow b}}$$

Now just solve the ratio for the unknown height:

Unknown Height = (Known Height/Shadow b) x Shadow a

Method 2: Measuring Height Using a 45-45-90 Degree Triangle

Tools Needed:

- Square piece of paper or cardstock
- Measuring tape or measured string

Optional:

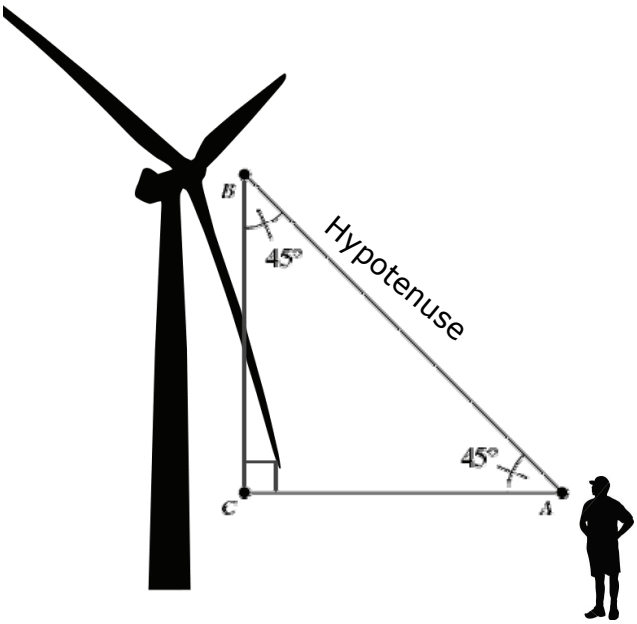
- A drinking straw
- Tape
- 1 short length of string
- A bolt or small weight

The Simple (but less accurate) Method:

1. Fold the square piece of paper in half so you have an isosceles right triangle (45-45-90 degrees).
2. Hold the triangle with one edge parallel to the ground and the hypotenuse angling up from your eye to the top of the wind turbine.
3. While sighting along the hypotenuse, move backwards or forward until the tip of the hypotenuse lines up with the top of the tower.
4. At this point, the height of the wind turbine is equal to the horizontal distance you stand from the wind turbine (plus the height from the ground to your eye).

For More Accuracy, Make a Home-made Clinometer:

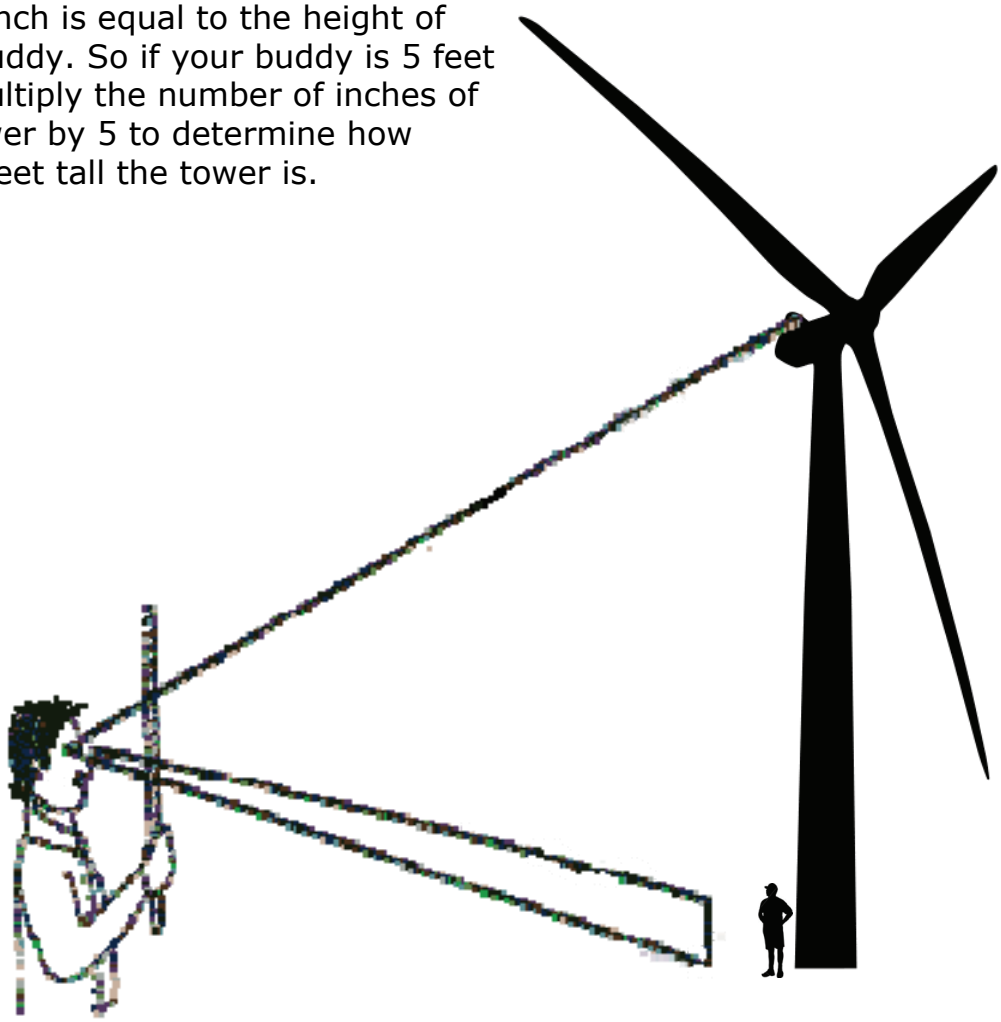
1. Fold the square piece of paper like step 1 above to make the 45-45-90 degree triangle.
2. Tape a drinking straw along the hypotenuse.
3. Attach the short string to the upper end of the hypotenuse and tie a weight to the bottom of the string. The weight should dangle a couple inches below the bottom corner of the triangle.
4. Follow the directions as above, but now you can look through the drinking straw to the top of the tower. When the weighted string is parallel to the vertical edge of the triangle, you can be sure that the bottom edge of the triangle is parallel to the ground.
5. The height of the tower will be equal to the horizontal distance you stand from the tower plus the distance from the ground to your eye.



Method 3: Measuring Height Using a Yardstick and a Buddy

For this measurement, you will need a yardstick and a partner who knows how tall he or she is. A calculator may help as well.

1. Have your buddy stand at the base of the tower
2. Holding the yardstick vertically, back up from the tree to the point where your buddy fills exactly one inch on the yard-stick.
3. Without moving the yardstick from that point, look from the bottom of the tower to the top of the tower. Note the number of inches that the turbine fills up.
4. Every inch is equal to the height of your buddy. So if your buddy is 5 feet tall, multiply the number of inches of the tower by 5 to determine how many feet tall the tower is.



For example: Your friend is 5 feet tall. After doing the procedure as described above, the wind turbine tower takes up 35 inches. How tall is the tower? 6 feet x 35 = 210 feet tall.

Sample Problems:

1. You measure the shadow of a wind turbine to be 164 feet and 4 inches. At the same time of day, the shadow cast by a 3-foot tall yardstick is 1.7 feet. How many feet tall is the wind turbine?

2. You measure your own shadow to be 8 feet long. Then you measure the shadow of a wind turbine to be 500 feet long. How tall is this wind turbine? (Hint—you'll need to know your own height).

3. Pretend you are 5 feet, 9 inches tall. You are using an isosceles right triangle (method 2) to find the height of a wind turbine. You line the tip of the hypotenuse up with the top of the tower and mark the ground where you stand. Sadly, you don't have a long enough tape measure to go from the base of the wind turbine to the mark where you stood. So, you measure your stride and find that every step you take is about 2.2 feet long. Next you pace off the distance from your mark where you stood to the base of the turbine. It takes 111 steps. About how tall is this wind turbine?

4. Pretend that your math teacher used to play in the NBA. He was good at dunking because he is 7 feet tall! Now he has asked you to find the height of a wind turbine tower. He stands at the base of the tower and gives you a yardstick. You back up until he takes up exactly one inch of the yardstick. At this point, the wind turbine fills 32.5 inches of the yardstick. How tall is this tower?

5. Which of these methods for determining height of a tower (method 1, 2, or 3) do you think is the most accurate? Why? Name one reason why each method might not be perfectly accurate.

Did You Know?

The tallest installed wind turbine is the Enercon E-126. The hub of this wind turbine is 135 meters off the ground! One meter is 3.28 feet, so that means this turbine is 442 feet high!!! Each blade is over 60 meters long, so the blade tips reach about 639 feet in the air! It is rated at 6 Megawatts. That's one big, powerful machine!